AIRCRAFT TRAILING VORTEX INSTABILITIES

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Abstract

A brief summary is given of the results of flight test studies of trailing vortices carried out at Texas A&M University. The instability of a pair of trailing vortices due to mutual interaction is fully discussed and theoretical predictions of the wavelength of the vortex oscillations that develop in the far wake of an airplane are compared with values determined from photographic records of the wake behavior of a DeHavilland Beaver DHC-2 aircraft. The different types of instability that can develop with single vortices are also considered, including the vortex bursting phenomenon that occurs with vortices that separate from the leading edges of highly swept wings at incidence. A technique for inducing earlier breakdown and dissipation of the vortices than would occur normally is described.

I. Introduction

Concurrently with the development of the Boeing 747 and other large airplanes, much effort has been devoted to research on aircraft wake turbulence, particularly for low altitude flight. This was because the regions of high velocity generated by the trailing vortices of such heavy aircraft are a source of potential danger to other aircraft that might penetrate them. On a calm day, the vortex wake of a heavy transport plane that had just landed or taken off would persist over the runway for several minutes and make it dangerous for any other aircraft to land. In fact several accidents to light aircraft have occurred due to the pilot flying inadvertently into the wake of a much heavier aircraft. To avoid such hazards, traffic controllers at airports must have some idea of the length of time it takes for the vortex wake of a large aircraft to dissipate and to become relatively harmless to other aircraft. Hence, from the point of view of safer flying, it is important to know as much as possible about the behavior of vortex wakes, their structure and the time they take to dissipate.

In 1970, an important symposium on the subject of Aircraft Wake Turbulence(1) was held in Seattle to draw attention to the need for more research to determine the characteristics and behavior of trailing vortices and to assess the potential danger they would present to aircraft at busy airports. The meeting, sponsored by Boeing Scientific Research Laboratories and the U.S. Air Force Office of Scientific Research, resulted in a number of valuable reports which have been published in the proceedings of that symposium. These collected papers provide a useful background of information on trailing vortices and give a fairly complete survey of the state of knowledge at that time as well as indicating where further work is required. More particularly, they show that there is a need for more data on trailing vortices obtained in actual flight. Some of the reports describe in very general terms the results of observations made of the characteristics of trailing vortices as revealed by smoke trails from smoke bombs mounted on towers alongside the runway. When there is a cross wind, the wake drifts across the tower and the presence of the trailing vortex is clearly shown by the vortical pattern formed. An alternative scheme is to build a large tufted screen and to let the vortex again drift across it. Both schemes, however, have the disadvantage that the presence of the tower in the one case and the supporting structure of the screen in the other can influence the trailing vortex and possibly cause it to change its form. For this reason, it is desirable to use techniques of observation in which the trailing vortices are not obstructed in any way. A flow visualization scheme in which the vortex cores are seeded by smoke issuing from smoke bombs suitably mounted on the wings of the vortex generating aircraft has proved to be very satisfactory. This method has been used extensively at the Flight Mechanics Laboratory at Texas A&M University and detailed observations have been made of vortex wakes for a range of flight conditions in calm and gusty weather.(2) Attention was mainly devoted to the study of trailing vortex instabilities. The results obtained are briefly summarized in this paper and, as will be shown, confirm some theoretical deductions regarding the unstable characteristics of aircraft trailing vortices.

II. Instability Of A Pair Of Vortices

The stability of a pair of trailing vortices was first considered by S. C. Crow(3) who was able to show that the vortices would become unstable under certain conditions. In his analysis, he assumed that the vortices were given a small wave-like disturbance and then investigated the effects of mutual interaction and self-induction on the ensuing motion. The velocity induced at a point of one vortex can be expressed in terms of integrals taken along the lengths of the vortices. The integral corresponding to the self-induced velocity of a vortex at a point on itself is, however, divergent and to obtain a finite answer, Crow had to use the 'cut-off integral' method which introduces an empirical factor in his analysis. This involved cutting off a length d of the vortex on each side of the point on it at which the self-induced velocity was to be calculated. The choice of a suitable value for d was made by applying the 'cut-off integral' technique to problems whose exact solutions are known, e.g., that of calculating the translational speed of a vortex ring. In this way, it was determined that a value $d = 0.321\ c$ would give the correct answer, c being the core diameter of the vortex. According to Spreiter and Sacks,(4) the trailing vortices behind an elliptic wing of total span s would be separated by a distance, $b = \frac{\pi}{4} s$, and each vortex would have a core diameter, $c = 0.197b$. Having chosen the appropriate value for d, Crow was then able to reduce the problem to an eigenvalue problem and to determine the stability characteristics of the trailing vortices. His analysis revealed that the most likely mode of instability of a pair of parallel line vortices would be in the form of symmetrical sinusoidal oscillations of the vortices in planes inclined at angles of ± 48° to the horizontal as sketched in Figure 1.

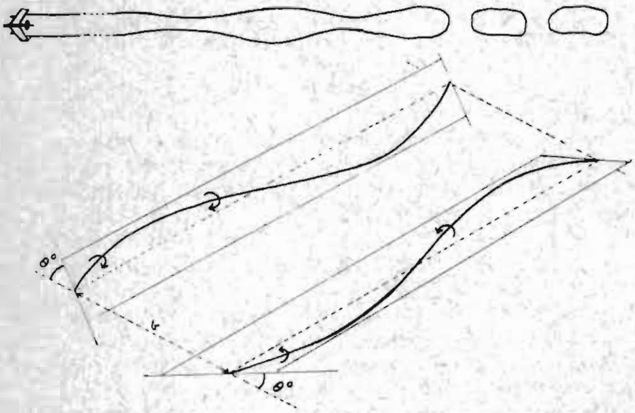


FIGURE 1. INSTABILITY OF A PAIR OF VORTICES

The wavelength of the oscillations was estimated to be 8.6 b (or 6.8s) and their rate of growth was found to be relatively slow. According to Ref. 3, the amplitude of the oscillation would increase by a factor e in time, $t = A_R b/C_L V_0$. For a B-47 airplane, travelling at $V_0 = 720$ f.p.s., Crow gives a value of 21 secs. for t which implies that in 60 secs. the amplitude of the vortex oscillations would be magnified 17.4 times. When the oscillations become sufficiently large, the vortices touch and combine to form a series of closed loops which eventually disperse. Although the structure and axial velocity distributions within the vortex cores are not taken into account in Crow's analysis, his general conclusions have been amply confirmed by flight data.(2)

In a recent paper, H. Chevalier(2) describes the results of a series of flight tests made to study the formation and dissipation of trailing vortices. The two aircraft used were a DeHavilland Beaver DHC-2 and a Beechcraft T-34B. Smoke bombs were attached below the wings of each aircraft near the tips and, when activated by the pilot, they seeded the cores of the trailing vortices with smoke as illustrated in Figure 2. The smoke trails produced were then photographed from above by a chase airplane at altitude and from the ground. Figure 3 is a photograph of the pair of trailing vortices generated by the DeHavilland Beaver taken from a following plane. The Crow type instability of the wake is clearly demonstrated. No measurements were made of the inclination of the planes of oscillation of the trailing vortices but they were observed to be about ± 45° to the horizontal and to be roughly in accord with Crow's prediction. Measurements of the wavelengths of the unstable motion of the wakes of the Beaver and T-34B aircraft were obtained from pictures of their trailing vortices taken from the ground.

(A) DEHAVILLAND BEAVER DHC-2

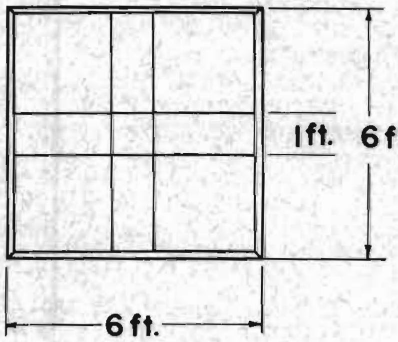
(B) BEECHCRAFT T-34B

FIGURE 2. SMOKE SEEDING OF VORTEX CORES

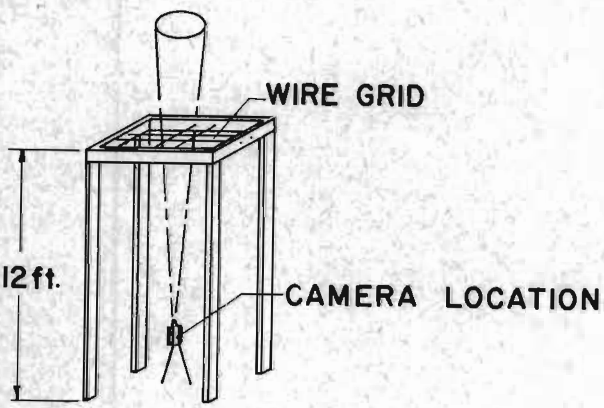
The wakes were photographed through a 6 ft. x 6 ft. wire grid mounted 10 ft. above the camera at ground level as shown in Figure 4. A clock mounted in its field of view and the frame speed of the movie camera were used to measure the time elapsed. The wire grid provided a reference coordinate system for measuring distances parallel to the ground and altitude. Values of height above the ground deduced from photographs of the airplane and its wake were checked against the airplane's altitude meter readings and the airplane's wing span was used to check measurements of distances.

Moving and still pictures of the trailing vortex wake were taken for straight and level flights at speeds of about 70, 80, 90, and 110 knots at an altitude of about 1000 ft. Most of the tests were made in the early morning when there was little atmospheric turbulence to influence wake stability and dissipation. For comparison purposes, however, some tests were carried out under various degrees of atmospheric turbulence and in slight cross winds of 3-10 knots. While the data obtained from each film record as given in Figure 5 show a certain amount of scatter, they do, however, indicate that the value of 6.8 for the wake length/span ratio predicted by Crow is approximately correct despite the limitations of his theory.

FIGURE 3. VORTEX INSTABILITY IN CALM AIR FLIGHT



(A) TOP OF GRID



(B) SIDE VIEW

FIGURE 4. SKETCH OF CAMERA AND GRID INSTALLATION

The main source of error in the measured values is probably due to the loss of altitude of the wake formation with time which was found to be about 200 ft. in 2 min. for the DeHavilland Beaver. The initial loss of altitude in the near wake was fairly rapid but the rate of descent decreased with increasing time. No allowance for this was made in the analysis of film records taken of the various flights. The main source of error in the theory could be due to the simplified representation of the vortex pair by line vortices and to the fact that no allowance was made for the structure of the finite vortex core and axial velocity distributions within the core. A number of workers have refined Crow's work and studied the stability of vortices with finite core radii and appropriate distributions of vorticity within the cores. Parks,(5) in particular, estimated that the wavelength of the most unstable symmetrical oscillations of a pair of finite core vortices would be 7.2 b which is somewhat lower than but of the same order as Crow's predicted most unstable long wavelength of 8.6 b. Improved theory also indicates that these vortices would oscillate in planes at angles of ±45° to the horizontal instead of ± 48° as deduced for simple line vortices by the 'cut-off' integral method. The most unstable short wave oscillation which has the maximum growth rate was estimated by Parks(5) to have a wavelength of 0.57 b as compared to the value 0.42 b deduced from Crow's analysis. In this connection, reference should also be made to the work of Widnall, Bliss and Zalay,(6) and that of Moore and Saffman.(7) Their investigations and that carried out by Parks substantially confirm the findings of Crow's theory.

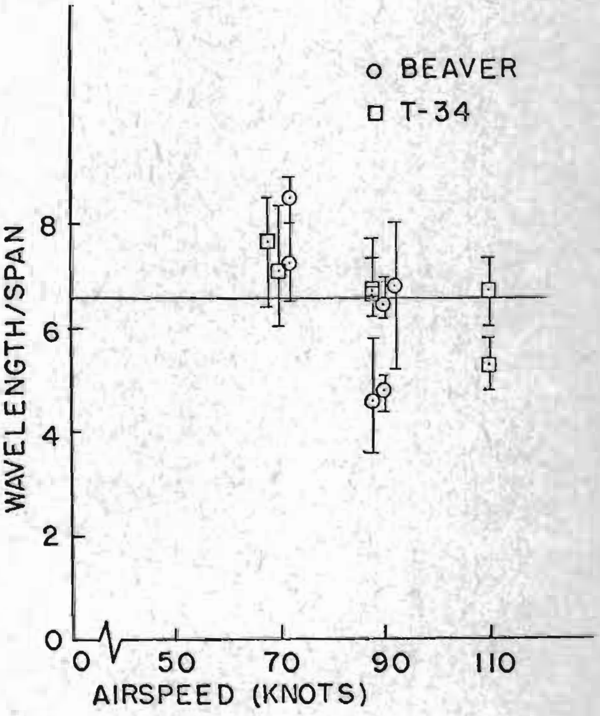


FIGURE 5. VORTEX INSTABILITY WAVELENGTH

Apart from the measurements of wavelengths made by H. Chevalier,(2) to which reference has already been made, there is hardly any quantitative data available. Many authors have taken photographs of vortex wakes but there appears to have been no systematic attempt to obtain accurate data on frequency and modes of oscillation from such photographs. However, from such observations as have been made, crude estimates of the wavelength of the predominant trailing vortex oscillation are in rough accord with theoretical predictions. Extensive wake studies(8) have been initiated by the Federal Aviation Administration on a wide variety of aircraft. The aircraft used included the Boeing 747, 707-300 and 727-100, the Douglas DC-8-63F, DC-8-33 and DC-9-10, the Lockheed C5A, the Convair 880 and the Learjet 24. Some measurements of the trailing vortex wake characteristics were made by aircraft specially instrumented for in-flight penetration of vortex wakes and by the tower 'fly-by'technique. In this way, quantitative data were obtained pertaining to the tangential flow generated round the vortex cores and core dimensions. Observations of axial flow were also made by the tower technique and it was found that whenever a vortex drifted across the tower, the vortex would entrain some of the smoke which would flow axially in both directions. This was presumed to be a consequence of the tower penetrating the low pressure central core of the drifting vortex and allowing higher pressure air from outside to enter. The greater part of their investigation, however, was concerned with the determination of the response of an instrumented aircraft when penetrating or passing near the wake of a heavier and larger aircraft. No data was given on vortex wavelengths and the stability of the trailing wakes was not specifically studied.

At the Royal Aircraft Establishment, Farnborough, Bisgood, Maltby and Dee(9) investigated the behavior of trailing vortices near the ground. They used an instrumented aircraft to probe the flow field induced by the wake of another aircraft and carried out response tests. With a Hunter aircraft flying near the ground they found no indication of the Crow type of instability. However, it was observed that occasionally one or both of the vortices broke down into a series of semi-circular arches with both ends perpendicular to the ground as shown in Figure 6. This suggests that each of the trailing vortices interacts with its image in the ground to produce an instability.

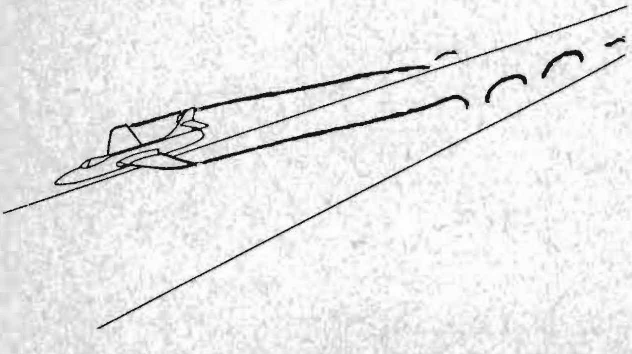


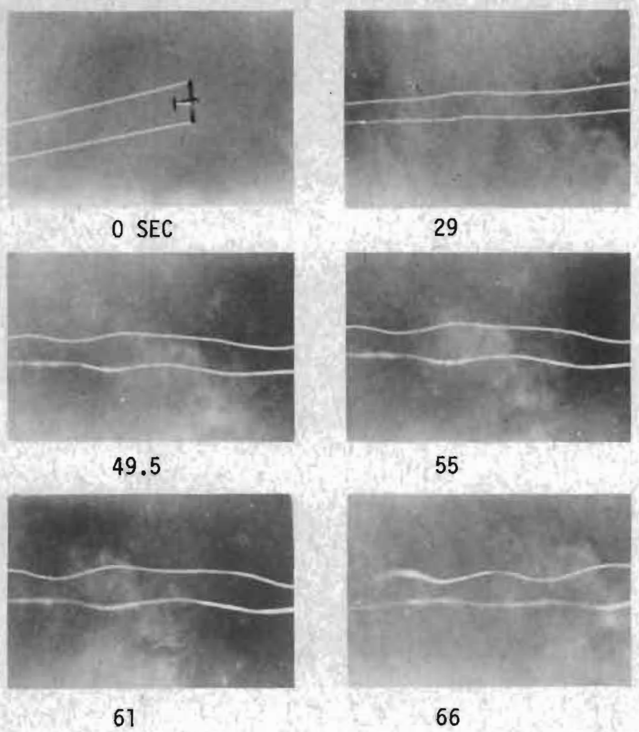
FIGURE 6. SKETCH OF VORTEX BREAKDOWN NEAR THE GROUND

A theoretical study of this particular phenomenon has been carried out by Rotta(10) who was able to show that the vortices were unstable to small disturbances and that the rate of growth of the amplitude of oscillation increased as the vortices approached nearer to the ground. When the distance above the ground is less than half the distance between the trailing vortices, ground effect becomes predominant and instability arises due to interaction between a trailing vortex and its image in the ground plane rather than between the pair of trailing vortices.

So far discussion has been mainly limited to the instability of the trailing vortices as a pair where, as has been mentioned previously, the vortices break down into a series of loops after a sufficient lapse of time. Figure 7, obtained by H. Chevalier,(2) shows the typical behavior and dispersion of the vortex wake of the DeHavilland Beaver aircraft when flying at an altitude of 1,000 ft. MacCready(11) has given a similar photographic record of the wake behind a B-47 and, in Ref. 9, the wake of a Comet aircraft is shown to behave in almost an identical manner. The initially parallel trailing vortices first oscillate in planes at about ± 45° to the horizontal and when the oscillations become large enough, the vortices touch and break down into a series of closed loops which eventually disperse. However, a detailed examination of the wake patterns of various aircraft reveal that it is possible to have single vortex breakdowns as well as the Crow type of instability under extremely calm air flying conditions in the early morning. With increasing time, generally 1-2 min, one type of breakdown appeared as a simple break in an individual vortex as shown in Figure 7. It was found that this type of instability could occur when the vortices were unstable as a pair and describing their usual wave formation. However, it was noted that, later in the morning, there were fewer simple breakdowns of an individual vortex but more vortex loops from the pair of trailing vortices. This is thought to be due to convection currents and small amounts of turbulence produced by the heat of the sun.

On several occasions, early morning fog or haze made it impossible to get good photographic records of wake patterns. However, by visual observation, it was found that this type of atmospheric condition had a pronounced effect on vortex dissipation. The vortices broke down very rapidly in the form of simple breaks and dissipated in less than 20 secs. while under calm clear air conditions they persisted for up to 2 mins. or more. These results clearly indicate that under atmospheric conditions of high humidity, the hazards due to wake turbulence would be greatly reduced. Similarly, with small increases in atmospheric turbulence caused by gusts or crosswinds, it was observed that sinusoidal wave formations appeared more quickly and more vortex rings were formed. In general, the effect was to reduce the total dissipation time of the wake. Further increases in atmospheric turbulence, such as would be produced by hot air currents under otherwise calm conditions resulted in vortex breakdown before the formation of vortex loops and in even shorter wake dissipation times.

(A) SIMPLE BREAKS



(B) VORTEX LOOPS

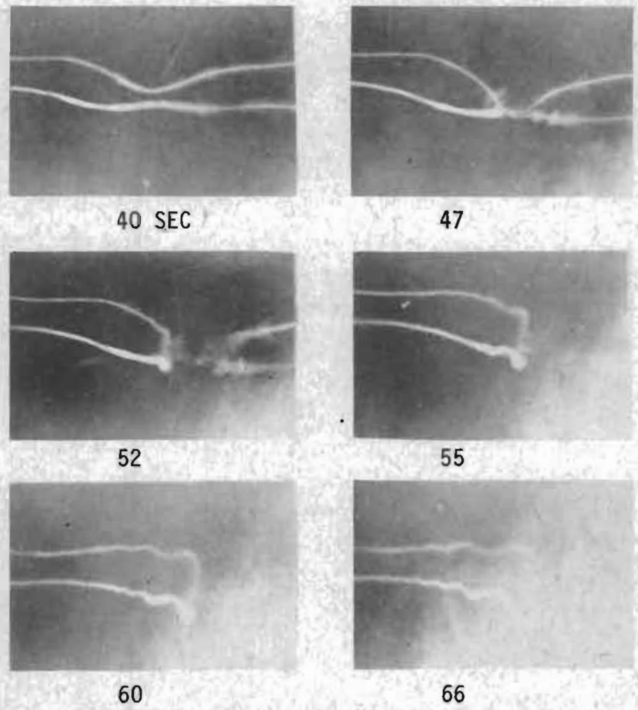
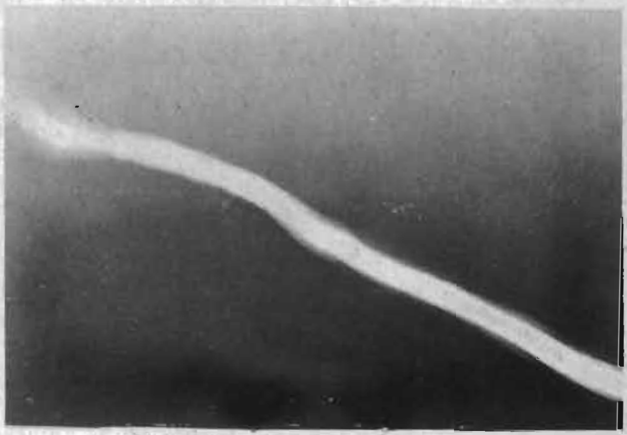


FIGURE 7. VORTEX WAKE DISPERSION PATTERNS

III. Single Vortex Instabilities

In Ref. 2, Chevalier draws attention to the fact that the instability or breakdown of a single vortex can be of two types. The first type is characterized by conical turbulent dispersion of the vortex core into the surrounding atmosphere as shown in Figure 8. The second type appears as a simple stretching of the vortex core until it completely disappears as also indicated. In the course of the flight tests made (more than 200), it was observed that the vortex core was always well defined before breakdown and that there was no noticeable movement of fluid particles out of the core. Also, there was apparently no entrainment of particles from the region surrounding the core. At all times, the flow within the core appeared to be calm and stable, indicating that the level of turbulence was generally low. Detailed measurements of the velocity and vorticity distributions were, however, not made. From the pictures taken of the vortex wake and general observation of its behavior from a chase airplane, it was concluded that there was little axial flow except at vortex breakdown when axial flow in both directions was observed. Wind tunnel and towing tank model tests, however, have shown that there can be axial flow in the vortex cores in the near wake a few chord lengths downstream of the vortex generating wing, probably induced by pressure gradients in the external flow.

(A) TURBULENT DISPERSION



(B) VORTEX STRETCHING

FIGURE 8. DISSIPATION OF A SINGLE VORTEX

Extensive experimental studies have been made of the stability of the vortices that separate from the sharp leading edges of highly swept low aspect ratio wings at incidence. Most investigators used wind tunnels or water tunnels (tanks) and observations of the wake vortices were mainly limited to the region near the generating wing, usually referred to as the near wake. Studies(9) have also been made in flight of the wake structure of the Handley Page, H.P 115, research aircraft which has a delta planform of low aspect ratio. (A=0.93) These were, however, of a preliminary nature and little information was obtained on the type of vortex breakdown and its location relative to the airplane.

Wind tunnel and water tunnel tests have revealed that leading edge separated vortices can become unstable in two different ways. In one type the core of the vortex, before it bursts, appears to swell into an axisymmetric form as if there were a stagnant bubble in the flow. In the second type, on the other hand, the vortex core suddenly spirals around itself and the flow becomes highly turbulent. Both types are shown in the remarkable picture taken by Lambourne(12) of flow over a delta wing at incidence in a water tunnel which is reproduced as Figure 9 of this paper. He also investigated these vortex instabilities in swirling water flow in a tube in which he was able to produce both forms of vortex bursting for study. From his observations of the development of the instabilities, he concluded that the spiral form could be regarded as due to some inherent asymmetry in the flow structure as usually occurs for leading edge separated vortices or as a consequence of the breakdown of the axisymmetric bubble formed in the core. The time history of the transient development of a breakdown is sketched in Figure 10 which is reproduced from Ref. 12.

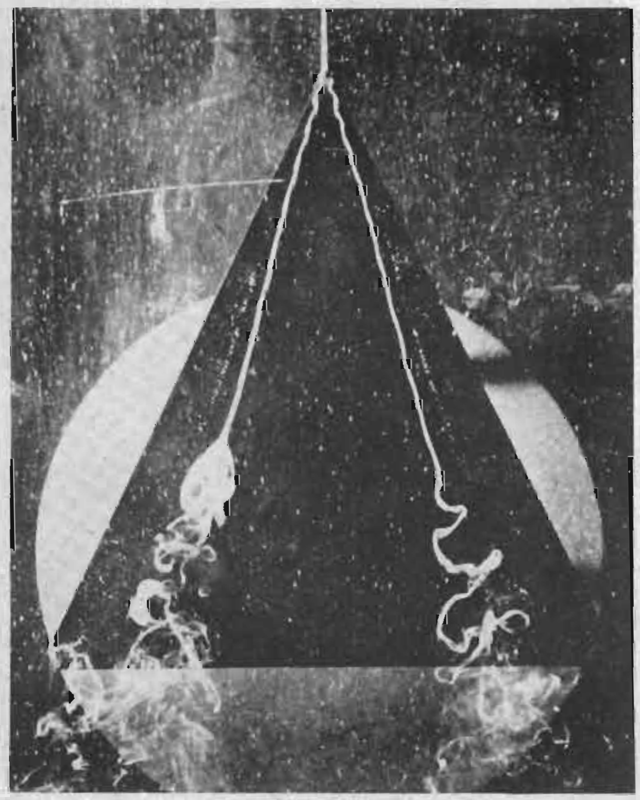


FIGURE 9. TWO TYPES OF VORTEX BREAKDOWN

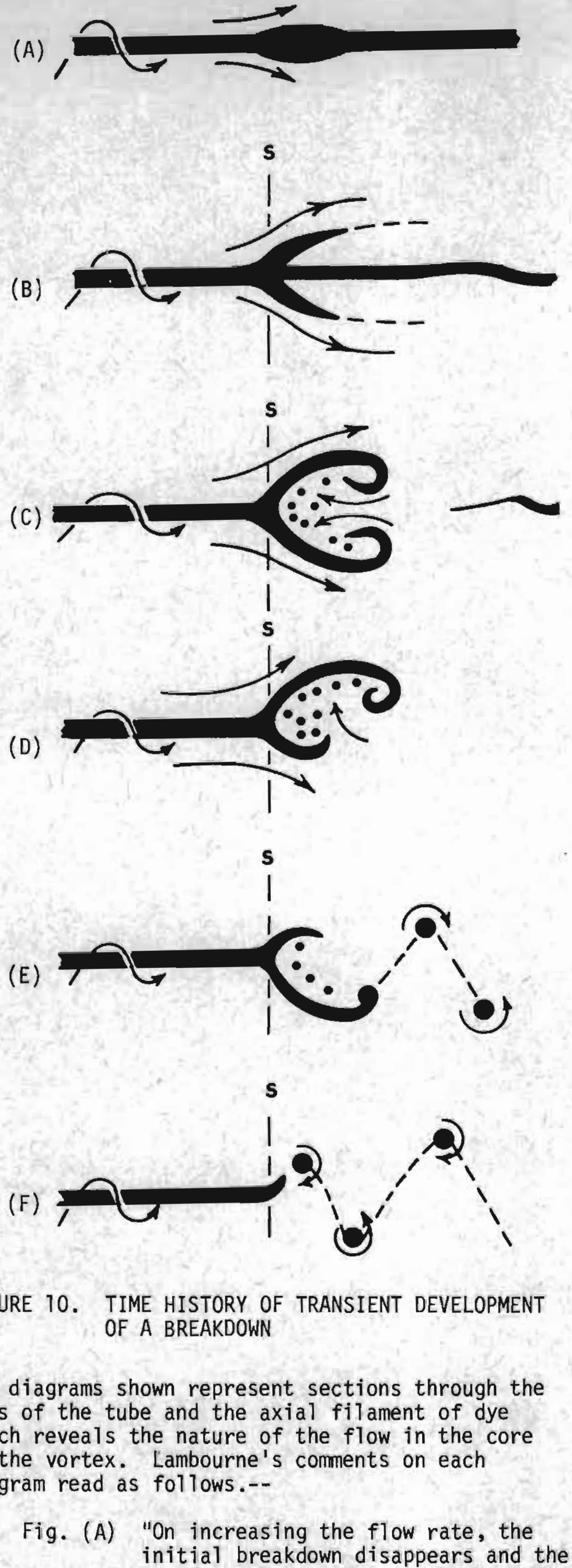


FIGURE 10. TIME HISTORY OF TRANSIENT DEVELOPMENT OF A BREAKDOWN

The diagrams shown represent sections through the axis of the tube and the axial filament of dye which reveals the nature of the flow in the core of the vortex. Lambourne's comments on each diagram read as follows.--

Fig. (A) "On increasing the flow rate, the initial breakdown disappears and the dye forms a narrow steady filament along the center of the tube. Then, on decreasing the flow rate, a thickening appears in the filament as shown in the diagram.

Fig. (B) The thickening rapidly develops into a surface of dye having the form of a wine-glass or tulip. At this stage, the dye in the central filament to the right of S is quite stationary, but to the left of S dye still flows along the axial filament and spreads over the surface of the tulip shape. This behavior is consistent with the presence of a core of fluid that has no longitudinal velocity component, and which is indicated in the diagram by the broken lines; it would seen likely that this body of fluid became stationary at the same time as the initial thickening of the dye filament was first observed.

Fig. (C) By the next stage, the dye shape has become re-entrant. The behavior of the remnant of dye filament to the right of S shows that fluid is now moving upstream within the tulip, and this combined with the movement of dye at the surface shows the presence of a vortex ring which has the basic axisymmetric arrangement.

Fig. (D) The onset of asymmetry marks the next stage. The circulatory, or vortex ring, flow within the tulip appears to cant to one side and to rotate and this leads to a kind of emptying process in which the trapped fluid is shed downstream along a spiral path. Observations of some transients suggest that spiralling first appears well downstream of point S and that the spiral character is propagated upstream until it reaches the vortex ring system. It is possible that this upstream propagation leads to the breakdown of the axisymmetric flow.

Fig. (E) The subsequent development was most difficult to follow but it appears that, with the fluid which was previously trapped now moving downstream, the dye forming the surface of the tulip becomes attenuated; the dye which has flowed along the axis now remains as a discrete filament which is convected downstream to give the final spiral form, Figure 10 (F), already described.

In short, although the details of the changeover remain obscure, the breakdown phenomenon appears initially in an axisymmetric form as a core of stationary and, later, circulatory flow which evolves to the final spiral arrangement. The observations thus suggest that the spiral form observable under steady conditions should be considered as a derivative of the axisymmetric form. Thus, in seeking a theoretical explanation of the phenomenon, it may be necessary, firstly to provide an explanation of the axisymmetric form and then to consider the stability of such a flow with the possibility of its degeneration to another form."

In wind tunnel or water tunnel studies of the leading edge separated vortices on a highly swept wing at high incidence, the spiral type of instability usually occurs near the wing's trailing edge. This is believed to be associated with the adverse pressure gradients that exist in the flow in the near wake. The effect of such a longitudinal pressure gradient on a simple vortex has been considered theoretically by Lambourne.(12) He was able to show that reversed axial flow could occur before the critical condition for vortex breakdown was reached but apparently no realistic solution could be found when the critical conditions was exceeded. This, he interpreted, as indicating that an imposed change of pressure gradient (or retardation of the external flow) greater than the critical value could lead to an unspecified drastic change of flow structure which would not be covered by his analysis.

Many other research workers have treated the problem of vortex instability theoretically and have offered various explanations for vortex breakdown. According to Landahl and Widnall(13), the instability mechanism is not now regarded as being the primary cause of breakdown and the main controversy at present is between those who believe it to be due to stagnation and those who think of it as a sudden transition as suggested by Benjamin.(14,15) He envisaged breakdown as a process similar to a hydraulic jump or a shock wave and this explanation is probably the one most favored at present. Landahl and Widnall(13) have given a simple qualitative treatment of the transition process using a 'one dimensional' model of a vortex and considering the effects of core energy changes on critical velocity and core area. They conclude, however, that vortex breakdown is probably a much more complicated phenomenon than Benjamin's transition model would suggest and propose that it should be considered more appropriately as a blocking or choking phenomenon.

The types of vortex breakdown that occur on leading edge separated vortices have not been observed in the far wake of a vortex generating aircraft under cruising conditions. It appears that neither the spiralling of the vortex core nor the axisymmetric swelling of the core can occur when there is no pressure gradient in the exterior flow. A careful examination of the wake pictures taken by Chevalier indicates that the breakdown of trailing vortices is much less violent and can be due to simple dispersion or vortex stretching. It is evident, however, that much research is required before the various types of breakdown and their causes are completely understood.

IV. Artificially Induced Instabilities

In order to reduce the potential danger to light aircraft flying into and out of busy airports, ways and means have been considered for causing more rapid dissipation of the strong trailing vortices created by heavier aircraft. It has been suggested that earlier breakdown of the vortices could be induced by imposing a disturbance at the particular frequency at which they become unstable according to Crow's theory. Ideally, one would like to produce such a disturbance in the wake without changing lift but, unfortunately, this was not feasible at the time the tests were made. Preliminary flight tests made with the DeHavilland Beaver during calm conditions with the elevator oscillated at the appropriate frequency showed a marked reduction in the time it took the vortices to dissipate. Additional testing was then done to determine the amplitude of the pitching oscillations of the airplane that would be required and the range of frequency for which dissipation time would be reduced. By oscillating the airplane's elevator, pitching oscillations of $\pm 2^{\circ}$ about a mean incidence of about 6° were induced at a speed of 80 knots. The frequencies of oscillation were 0.5, 0.25 and 0.125 HZ respectively, 0.25 HZ being the estimated critical frequency for the airplane. A comparison of Figs. 11 and 12 for an imposed $\pm 2^{\circ}$ oscillation at a frequency of 0.25 HZ shows that the dissipation time would be reduced to about half that required for dispersion in steady flight at the same speed in calm air. It was also found that increasing the amplitude of the pitching oscillation further reduced the dissipation time. Oscillations of smaller amplitude thatn $\pm 2^{\circ}$ were difficult to produce manually but from the tests made, it appeared that even the smallest oscillation at the right frequency had a beneficial effect. The results obtained for oscillations at 0.5 HZ also showed a reduction in wake dissipation time but only about half as much as for the 0.25 HZ case.

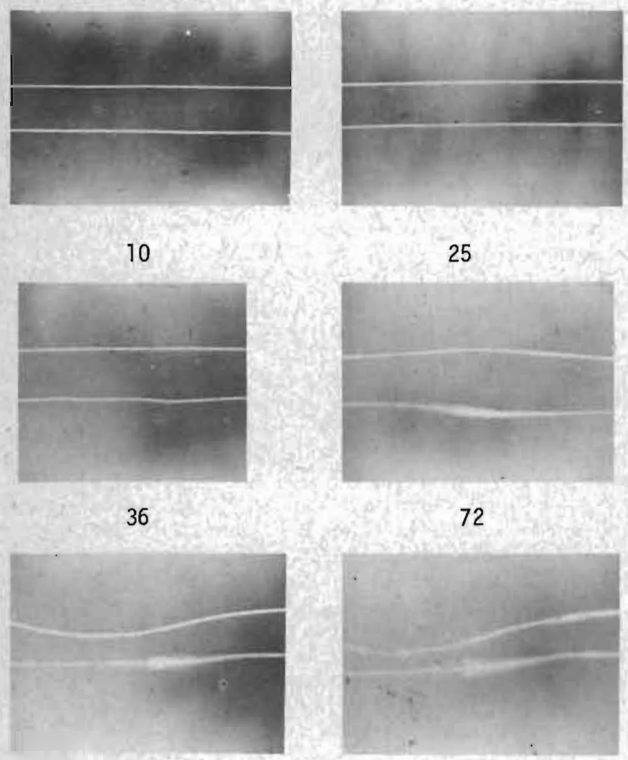

10 25
36 72
85 97

FIGURE 11. VORTEX CORE DISSIPATION IN STEADY FLIGHT

A puzzling feature of these results is that though the intention was to induce the usual wavy form of instability of the pair of vortices to occur earlier, the type of instability produced corresponds to periodic vortex bursting as shown in Figure 12 at roughly the same wavelength. Enlarged views of the vortex bursts are shown in Figure 13, while Figure 14 shows the location of the vortex bursts along the wake when the airplane is describing pitching oscillations. In all cases, the initial burst coincided with the minimum angle of attack.

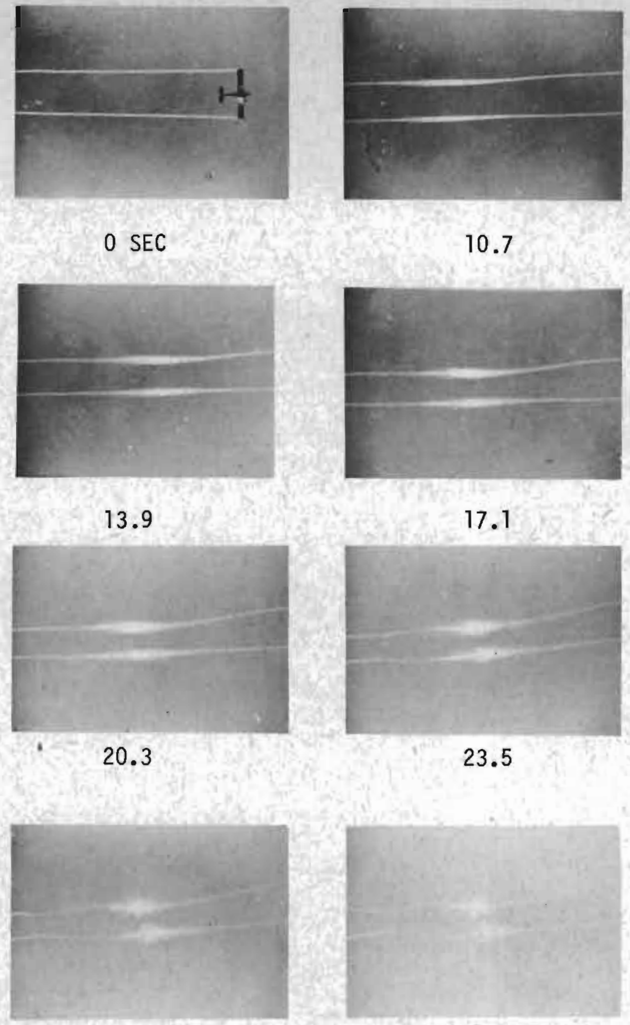

0 SEC 10.7
13.9 17.1
20.3 23.5
26.7 29.9

FIGURE 12. VORTEX CORE DISSIPATION WITH AIRPLANE DESCRIBING PITCHING OSCILLATIONS OF $\pm 2^{\circ}$ AMPLITUDE

From these results it would appear that a small symmetrical oscillatory disturbance of appropriate frequency which involved shedding of transverse vortices into the wake would markedly reduce dissipation time. It was found that oscillations in roll or yaw at the critical frequency did not have the desired effect. Tests in which symmetrical disturbances would be imposed on the wake without changing the total lift on the airplane are planned but as yet no results are available.

V. Concluding Remarks

It is clear from the evidence given that much more research will be required before a complete understanding of all facets of the vortex bursting phenomenon is achieved. The various types of instability that can occur require further detailed investigation. From the point of view of its practical importance to aviation, particular attention should be given to studies of methods of inducing wake instability to reduce the time it takes the trailing vortices to disperse.

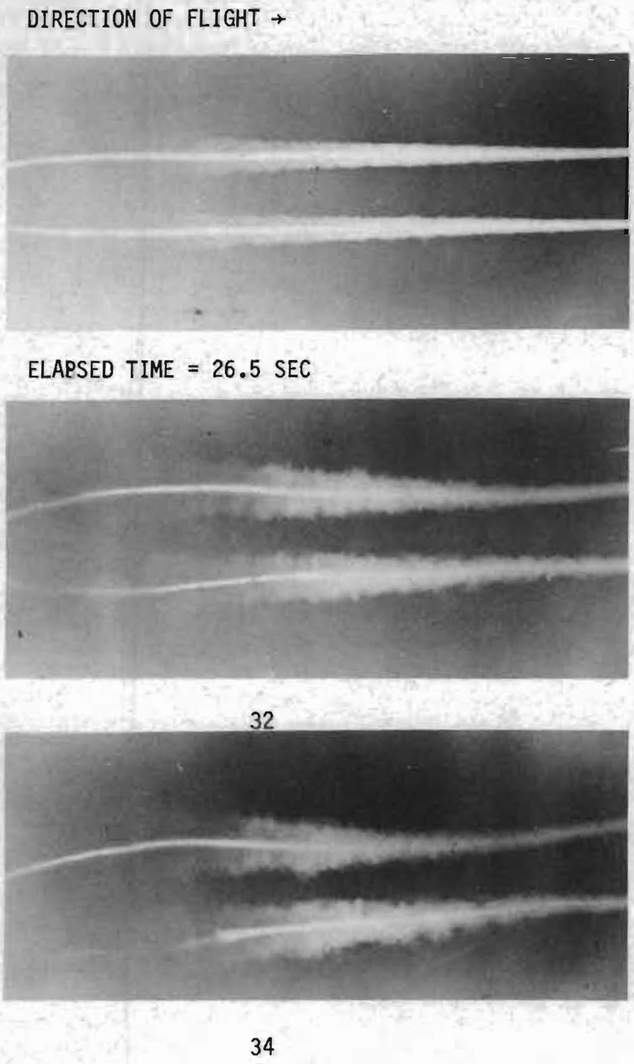


FIGURE 13. ENLARGED VIEWS OF VORTEX BREAKDOWN

FIGURE 14. VORTEX BURSTING FOR OSCILLATIONS OF THE DHC-2 AIRPLANE, FREQUENCY 0.25HZ; ANGLE OF ATTACK CHANGE ± 2°.

VI. Acknowledgment

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DISCUSSION

A. Das (DFVLR-Braunschweig, Germany): In some of the flight tests showing vortex instabilities or vortex bursting the airplane was describing pitching oscillations. In that case a fluctuation of circulation $\pm \Delta\Gamma$ would be initiated, causing shedding of transverse vortices from the wing trailing edge, these having each time opposite rotations. These vortices can significantly interfere with the main vortex field and bring their own contributions. It would be interesting to assess how strong these transverse vortices can be and how much influence they can have on vortex instability.

W.P. Jones and H.L. Chevalier: The question raised by Dr. Das is one that the authors have already considered themselves as it is obvious that the shedded transverse vortices must have a significant interference effect on the main trailing vortices. We agree that it would be interesting to assess how these transverse vortices could influence the trailing vortex instabilities and this should be looked into in the near future.

G.M. Lilley (University of Southampton, U.K.): I would be pleased if you tell us whether you were able to observe in the photographs you obtained of the vortex patterns in flight under very calm atmospheric conditions any axial motions in the vortex cores far downstream of the aircraft. Also did you observe if the turbulence in the vortex cores was very much reduced with distance downstream?

W.P. Jones and H.L. Chevalier: In response to this question, we did not observe any axial motions in the core far downstream of the aircraft. However, this does not preclude an axial velocity. A small axial velocity would be difficult to detect due to the relative motion of the chase airplane when we were near the vortex. Axial motions would also be difficult to detect from ground observations due to the large distances involved.

We did not detect from visual observations any significant turbulence within the vortex cores at any point along the vortex length. If anything the cores appear to be extremely well organized with no apparent turbulence until a dissipation or core rupture occurred.

V.L. Marshall (British Aircraft Corporation, Weybridge, U.K.): I am interested in the flow phenomena referred to as "Vortex Bursting" as a result of the cyclic aircraft motion. Fig. 13 shows that a rapid dispersion of some of the vortex core has taken place, yet the remainder of the core can still be seen penetrating the turbulent region. After that, the cores seem almost to rejuvenate themselves, although the presence of these "bursts" are to eventually lead to vortex breakdown. Can an explanation be provided of this phenomena shown to us today?

W.P. Jones and H.L. Chevalier: The vortex bursting shown in Figure 13 is a consequence of the core dissipation upstream and downstream of the region shown in this Figure. From visual observations and other photographs in the report the core first appears to dissipate between the points where the bursting occurs. Upstream of the burst, the material captured within the core appears to move aft and flow over the initial core, similar to that of putting a sock over one's foot. From the downstream end the dissipation appears in a form such that the smoke particles flow upstream within the core; however, these particles appear to stay within the original core geometry. Thus the core seems almost to rejuvenate itself in the photographs. In our opinion this rejuvenation is simply an increase in the smoke particle density within the core near the bursting region and does not represent a rejuvenation of the core strength.